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TEMPERATURE FLUCTUATIONS IN A DISPERSE MEDIUM

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Large-scale temperature fluctuations in a thermally nonuniform disperse medium are analyzed by the methods of the thermodynamics of irreversible processes. Calculated results are compared with experimental data.

By participating in motions of various scales, particles of a thermally nonuniform disperse medium can "transport temperature" as an inert scalar admixture. Consequently, in certain regions of the system local large scale temperature fluctuations will arise with intensities appreciably greater than the level of equilibrium thermal agitation. Convective heat transfer between particles and the continuous medium must affect the dissipation of these fluctuations. The damping of large-scale fluctuations can be analyzed within the framework of the thermodynamics of irreversible processes (TIP), the thermodynamic theory of which was developed in [1] and discussed in detail in [2]. The theory was applied to hydrodynamic fluctuations in [3, 4].

The correlation function of temperature fluctuations T' can be written in the form

$$\varphi(t) = \varphi(t-t') = \langle T'(0)\,T'(t)\rangle = \langle T'(t)\,\overline{T'(t)}\rangle = \lim \frac{1}{\tilde{T}}\int_0^{\tilde{T}} T'(t)\,T'(t')\,dt',\quad \tilde{T}\to\infty, \tag{1}$$

where $\langle\ldots\rangle$ denotes probability averaging of all possible values of T' at times t and t'. $\overline{T'}$ is the average value of T' for $t > 0$ under the condition that this quantity had a given value T' at $t = 0$. Thus $\varphi(t)$ takes account of the previous history of the system from $t = -\infty$ to $t = 0$.

We consider the damping of temperature fluctuations T' of a continuous medium and T_1' of particles by TIP methods [1, 2, 5]. The phenomenological equation for them can be written in the form [2]

$$\dot{\mathbf{x}} = -\mathbf{M}\mathbf{x},\quad \mathbf{x} = \begin{pmatrix} T' \\ T_1' \end{pmatrix}. \tag{2}$$

The matrix **M** is evaluated in [6]:

$$\mathbf{M} = \begin{pmatrix} \alpha_0 C^{-1} & -\alpha_0 C^{-1} \\ -\alpha_0 C_1^{-1} & \alpha_0 C_1^{-1} \end{pmatrix},\qquad \alpha_0 C^{-1} = \frac{6\alpha(1-\varepsilon)}{d\rho c\varepsilon},\qquad \alpha_0 C_1^{-1} = \frac{6\alpha}{d\rho_1 c_1}, \tag{3}$$

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where $\alpha_0 = 6\alpha(1-\varepsilon)/d$ is the heat-transfer coefficient between liquid and particles per unit volume of the mixture. The dynamic equation for the correlation matrix φ is obtained from (2) by replacing x by φ:

$$\dot{\varphi} = -\mathbf{M}\varphi, \quad \varphi = \begin{pmatrix} \langle T'(t)\,T'(0)\rangle & \langle T'(t)\,T_1'(0)\rangle \\ \langle T_1'(t)\,T(0)\rangle & \langle T_1'(t)\,T_1'(0)\rangle \end{pmatrix}. \tag{4}$$

The solution of this equation can be written in the form

$$\varphi(t) = e^{-\mathbf{M}t}\varphi(0). \tag{5}$$

Taking account of (3), Eq. (4) for the correlation function of the particle temperature fluctuations has the form

$$\dot{\varphi}_{22} = -\frac{6\alpha}{d\rho_1 c_1}(\varphi_{22} - \varphi_{12}). \tag{6}$$

If $\varphi_{12} \ll \varphi_{22}$,

$$\varphi_{22} = \varphi_{22}(0)\,e^{-\frac{6\alpha}{d\rho_1 c_1}}. \tag{7}$$

Taking account of φ_{12} complicates the problem considerably. This is investigated in detail in [7]. In an adiabatic system the attenuation of correlations will be characterized solely by the relaxation time τ [6]:

$$\varphi = \varphi(0)\,e^{-t/\tau}, \tag{8}$$

where

$$\tau^{-1} = \mathrm{Sp}\,\mathbf{M} = \frac{6\alpha}{d}\left(\frac{1}{\rho_1 c_1} + \frac{1-\varepsilon}{\varepsilon}\,\frac{1}{\rho c}\right). \tag{9}$$

The relation $T_1' = -C/C_1 T'$ for adiabatic conditions was used in deriving (8) and (9).

We now consider quasistationary temperature fluctuations relative to a given distribution of the average $\langle T\rangle$ temperature of a disperse system. This problem was studied in detail in [5, 8, 9] for a single-phase medium. It was shown that the behavior of a fluctuating temperature field is closely related to the properties of the local potential. For a disperse system we write it in the form

$$L_T = \int_0^\infty \int_V \left[\frac{1}{2}(\nabla T)^2 + \frac{1}{k_0}T\,\frac{\partial^2\langle T\rangle}{\partial t^2} + \frac{k_1}{k_0}T\,\frac{\partial\langle T\rangle}{\partial t}\right] dt\,dV. \tag{10}$$

By varying (10) with respect to T for a fixed average temperature distribution, and then setting $T = \langle T\rangle$, we obtain the hyperbolic heat-conduction equation for a disperse medium

$$\frac{\partial^2 T}{\partial t^2} + k_1\frac{\partial T}{\partial t} = k_0\nabla^2 T. \tag{11}$$

This equation (in the more general case, taking account of the relaxation of the heat flux) was derived by statistical methods in [10], and by the TIP relaxation formalism in [11]. The coefficients k_1 and k_0 in (11) are equal, respectively, to

$$k_1 = \frac{C + C_1}{C\tau_1}, \quad k_0 = \frac{\lambda^*}{C\tau_1}, \quad \tau_1 = \frac{\rho_1 c_1 d}{6\alpha}. \tag{12}$$

The change of the fluctuating component will also be described by Eq. (11):

$$\frac{\partial^2 T'}{\partial t^2} + k_1\frac{\partial T'}{\partial t} = k_0\nabla^2 T. \tag{13}$$

By generalizing the definition of the correlation function (1) it is easy to derive from (13) the dynamic equation for $\varphi(\mathbf{r}, t) = \langle T'(\mathbf{r}, t)T'((0, 0)\rangle$. To do this it is necessary purely formally to replace T' by φ in (13) [1, 4]:

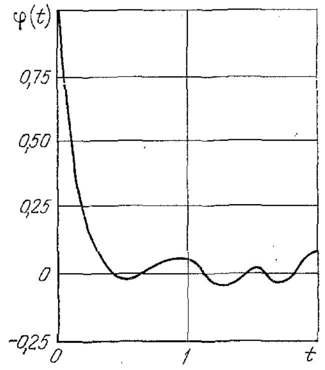

Fig. 1. Correlation function of temperature fluctuations in a fluidized bed.

$$\frac{\partial^2 \varphi}{\partial t^2} + k_1 \frac{\partial \varphi}{\partial t} = k_0 \nabla^2 \varphi. \tag{14}$$

We note that (14) is the Euler–Lagrange equation for the local potential L_T (10) in which the temperature T is replaced by φ:

$$L_\varphi = \int_0^\infty \int_V \left[\frac{1}{2} (\nabla \varphi)^2 + \frac{1}{k_0} \varphi \frac{\partial^2 \varphi^{(0)}}{\partial t^2} + \frac{k_1}{k_0} \varphi \frac{\partial \varphi^{(0)}}{\partial t} \right] dt dV; \tag{15}$$

$\varphi^{(0)}$ is not varied. Functional (15) for the construction of various approximate solutions of Eq. (4) was studied in [12].

We investigate the single-point time correlation function of temperature fluctuations by using Eq. (14). We write $\varphi(t, \mathbf{r})$ in the form

$$\varphi(t, \mathbf{r}) = f(t)\theta(\mathbf{r}). \tag{16}$$

Substituting (16) into (15) and integrating over the volume, we obtain

$$L_\varphi = \int_0^\infty \left(\frac{1}{2} Af^2 + \frac{B}{k_0} f \frac{\partial^2 f^{(0)}}{\partial t^2} + \frac{k_1 B}{k_0} f \frac{\partial f^{(0)}}{\partial t} \right) dt, \tag{17}$$

where

$$A = \int_V [\nabla \theta(r)]^2 dV; \; B = \int_V \theta^2 dV; \tag{18}$$

$f^{(0)}$ is not varied. Varying (17) with respect to f and setting $f^{(0)} = f$, we have

$$\frac{\partial^2 f}{\partial t^2} + k_1 \frac{\partial f}{\partial t} + \frac{A}{B} k_0 f = 0. \tag{19}$$

The solution of this equation has the form

$$f = e^{-\frac{1}{2} k_1 t} (\bar{c}_1 \cos \omega t + \bar{c}_2 \sin \omega t), \tag{20}$$

where

$$\omega = \sqrt{\frac{1}{4} k_1^2 - \frac{A}{B} k_0}. \tag{21}$$

Determining $\bar{c}_1$ and $\bar{c}_2$ from the system

$$\bar{c}_1 = f(0),\quad \dot{f}(0) = -\frac{1}{2}k_1\bar{c}_1 + \omega\bar{c}_2,\quad \bar{c}_2 = \frac{k_1 f(0)}{\omega}$$

and taking account of (16), we have finally

$$\bar{\varphi}(t) = e^{-\frac{1}{2}k_1|t|}\left(\cos\omega t + \frac{k_1}{2\omega}\sin\omega|t|\right). \tag{22}$$

Here $\bar{\varphi}(t) = \frac{\varphi(\mathbf{r},\ t)}{\langle T'(\mathbf{r},\ 0)^2\rangle}$, $\langle T'(\mathbf{r},\ 0)^2\rangle = f(0)\,\theta(\mathbf{r})$ is the mean-square temperature fluctuation at point **r**.

Expanding (22) in a Maclaurin series and retaining second-order terms, we determine the time scale of the correlations by the relation [13]

$$\frac{1}{\tau_E^2} = -\frac{1}{2}\left(\frac{\partial^2\bar{\varphi}}{\partial t^2}\right)_{t=0} = \frac{1}{2}\left[\omega^2 + \left(\frac{k_1}{2}\right)^2\right]. \tag{23}$$

The quantity τ_E is a measure of the most rapid changes of temperature fluctuations.

Temperature fluctuations were experimentally investigated in a fluidized bed [14]. Small Alundum beads were liquefied by water in an annular channel with an inner wall which could be heated. Temperature fluctuations at various points in the bed were measured with a thermocouple probe, and after amplification were recorded by a loop oscillograph. The oscillograms were processed on an F-001 analog–digital converter. Figure 1 shows a typical correlation function calculated from the experimental data on a Minsk-22 computer. It is clear that it is approximated by Eq. (22).

We present some estimates. The time scale of correlations for the data shown in Fig. 1 is $\tau_E = 0.074$ sec, and the frequency of oscillations is $\omega \sim 7.6\ \mathrm{sec}^{-1}$. The damping constant of correlations calculated from (23) is $k_1 = 35\ \mathrm{sec}^{-1}$. We used this value and Eqs. (3) and (12) to calculate the dimensionless heat-transfer coefficient $\mathrm{Nu} = \alpha d/\lambda = 48$. The L. K. Vasanov dimensionless relation [15] gives

$$\mathrm{Nu} = 0.35\,\mathrm{Re}^{0.8}\mathrm{Pr}^{0.33} = 61,$$

while that of Franz gives

$$\mathrm{Nu} = 0.006\,\mathrm{Re}^{1.3}\mathrm{Pr}^{0.33} = 51.$$

The following experimental and tabulated data were used in the calculations: $w = 0.187$ m/sec, $\varepsilon = 0.9$, $d = 1.38 \times 10^{-3}$ m, $\nu = 0.805 \times 10^{-6}\ \mathrm{m^2/sec}$, $\lambda = 0.62$ W/m·°K, Pr = 5.42, T = 303°K, $\rho_1 = 3590\ \mathrm{kg/m^3}$, $\rho = 998\ \mathrm{kg/m^3}$, $c_1 = 0.8$ kJ/kg·°K, c = 4.17 kJ/kg·°K.

It should be noted that high porosities $\varepsilon \sim 0.9$ were used in the experiments for purely technical reasons. Under these conditions the dimensionless relations can be employed only to estimate the order of magnitude of Nu. Nevertheless, the results obtained give us confidence that the theoretical model of correlations of temperature fluctuations in a disperse medium represents the basic features of large scale fluctuations.

NOTATION

t, time; **r**, coordinates; $\varphi(t)$, correlation function; $x = T'/T_1'$, temperature fluctuations; C, C_1, heat capacities of liquid and particles per unit volume of mixture; M, α_0, quantities defined in (3); α, heat-transfer coefficient; ε, porosity; d, particle diameter; ρ_1, ρ, densities of particle material and liquid; c, c_1, specific heats; φ, correlation matrix; τ, relaxation time of system; τ_1, particle relaxation time; λ^*, thermal conductivity of disperse medium; λ, thermal conductivity of liquid; V, volume of system; L_T, L_φ, local potentials: k_1, k_0, coefficients defined in (12); T, temperature; A, B, coefficients defined in (18); ω, frequency; τ_E, time scale of correlations; Re, Reynolds number; Pr, Prandtl number; Nu, Nusselt number; w, rate of filtration; ν, kinematic viscosity. Subscript 1 refers to dispersed phase; a dot denotes the time derivative.

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HYDRAULIC DRAG DUE TO DIVISION OF A STREAM OF FLUID INTO TWO PARALLEL CHANNELS WITH AN ARBITRARY RATIO OF FLOW RATES

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An expression is derived for the hydraulic drag and results of calculations are compared with experimental data.

The magnitude of the hydraulic drag at the inlet due to local separations of the stream during transfer of a liquid (or gas) from one channel to another can in many cases be found in [1]. As a rule, formulas recommended on the basis of experimental data are valid when the total flow rate remains constant during transfer from one channel to another. The theoretical solution obtained for straight channels of uniform cross-sectional area [2] and confirmed by results of experiments is valid only under that condition. There are semiempirical approximate relations available for determining the hydraulic losses which occur when a separate jet of fluid flows out of a stream (or into a stream) through a lateral channel at a given rate, at a given angle, and across a given area [1]. These formulas are, however, not sufficiently accurate for the simpler limiting cases such as, e.g., a zero exit angle or a zero flow rate through the lateral channel.

Here will be presented a theoretical solution to the problem, in the one-dimensional formulation, for determining the loss of total pressure due to entrance of a stream into a straight channel of uniform cross section from another one with a larger cross section. The smaller channel is completely inside the larger one and it takes up some arbitrary fraction of the total fluid flowing through the larger one (Fig. 1). A fluid here will include gases as well, but the effects of compressibility will be disregarded (Mach number $N_{Ma} \ll 1$). The hypothetical streamline along which the stream divides is indicated by dashes. The cross-sectional area of the stream, the pressure, the velocity, and the density of the fluid in channel 1 under steady state conditions (section 1-2) will be denoted as F_1, p_1, u_1, and ρ_1, respectively, and the corresponding parameters in channel 2 as F_2, p_2, u_2, ρ_2, respectively. In the segment of the initial stream in section 0–0 which subsequently enters channel 1 we denote the corresponding parameters as F_{01}, p_{01}, u_{01}, ρ_{01}; in the segment of the initial stream in this same section 0–0 which subsequently enters channel 2 we denote the corresponding parameters as F_{02}, p_{02}, u_{02}, ρ_{02}. The pressure, the velocity, and the density are assumed to be uniform within each thus defined segment of the stream cross section. The thickness of the layer between dividing stream segments is assumed to be zero,

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